

Fermion Propagator in Quenched QED3 in the Light of the Landau-Khalatnikov-Fradkin Transformation

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We study the gauge dependence of the fermion propagator in quenched QED3, with and without dynamical symmetry breaking, in the light of its Landau-Khalatnikov-Fradkin transformation (LKFT). In the former case, starting with the massive bare propagator in the Landau gauge, we obtain non perturbative propagator in an arbitrary covariant gauge. Carrying out a perturbative expansion of this result, it yields correct wavefunction renormalization and the mass function up to the terms independent of the gauge parameter. Also, we obtain valuable information for the higher order perturbative expansion of the propagator. As for the case of dynamical chiral symmetry breaking, we start by approximating the numerical solution in Landau gauge in the rainbow approximation in terms of analytic functions. We then use LKFT to obtain the dynamically generated fermion propagator in an arbitrary covariant gauge. We find that the results obtained have all the required qualitative features. We also go beyond the rainbow and encounter similar desirable qualitative features.

1. Introduction

Quantum electrodynamics in a plane (QED3) continues to attract attention both in the field of super-conductivity, e.g., [1], where it has been used in the study of high T_c super-conductors, as well as in the realm of dynamical mass generation (DMG) where the numerical findings on the lattice and the results obtained by employing Schwinger-Dyson equations (SDE), [2], are yet to arrive at a final consensus.

In the context of the SDE, the full fermion propagator (FP) is related to the full photon propagator (PP) and the full fermion-boson vertex (FBV), which obey their own SDEs. The study of the DMG is related to the knowledge of the FP. Quenched approximation consists of neglecting fermion loops. Within the realm of this approximation, we still need an *ansatz* for the FBV to study the FP. In the rainbow approximation, the FP equation decouples also from that of the FBV. The corresponding SDE gets even more simplified and one can extract the key features

of a dynamically generated FP. However, it is known that the bare vertex violates gauge invariance. In order to improve our approximations, we must impose gauge invariance constraints on the FBV, like Ward-Green-Takahashi identity (WGTI) [3], the Nielsen identities (NI) [5] and the Landau-Khalatnikov-Fradkin transformations (LKFT). Perturbation theory is the only known scheme where all of these identities and the gauge independence of physical observables can be achieved at every order of approximation. Therefore, we probably stand our best chance to achieve these features also non perturbatively if we construct a FBV which reduces to its perturbative counterpart in the weak coupling regime. This fact was recently exploited for the construction of the FBV in quenched QED3 by the authors [7]. A test of such constructions is to study the resulting SDE for various gauges, going in small steps of the gauge parameter away from the Landau gauge. It is prohibitively difficult to be able to compute the result for an arbitrarily large value of the gauge parameter especially if a so-

plicated form of the three point interaction is taken into account, [4]. Fortunately, one of the gauge invariance constraints, namely the LKFT, can help us to circumvent this problem.

The LKFT of the Green functions describe the specific manner in which these functions transform under a variation of gauge. These transformations are non perturbative in nature, and they are better described in coordinate space. The LKFT provide us with a mechanism to study the gauge dependence of the FP starting from its value in the Landau gauge. Both in the perturbative or non perturbative calculations, the knowledge of the gauge dependence of the Green functions is useful and the LKFT can help us to achieve this goal.

This work is organized as follows : In next section we recall how to perform the LKFT for the FP. Sect. 3 is devoted to the non perturbative FP obtained from the LKFT of the lowest order FP. We find valuable information in the perturbative expansion of this result. Sect. 4 is devoted to the LKFT of a dynamically generated FP obtained in Landau gauge with the bare vertex. We also extend these studies to the case of a solution for the SDE considering the full FBV (hereafter we will use the notation FV for the full vertex and BV for the bare one). Sect. 5 contains the numerical findings. Finally in Sect. 6 we present our conclusions and outlook.

2. LKFT : The Procedure

We start by putting forward the definitions and notations we shall use along the way [8,9]. We write out the FP in Euclidean momentum and coordinate spaces, respectively, in its most general form as :

$$S(p; \xi) = \frac{F(p; \xi)}{i\not{p} - \mathcal{M}(p; \xi)}, \quad (1)$$

$$S(x; \xi) = \not{x}X(x; \xi) + Y(x; \xi). \quad (2)$$

$F(p; \xi)$ is generally referred to as the wavefunction renormalization, whereas $\mathcal{M}(p; \xi)$ as the mass function. The above expressions are related through the following Fourier transformations

$$S(p; \xi) = \int d^3x e^{ip \cdot x} S(x; \xi), \quad (3)$$

$$S(x; \xi) = \int \frac{d^3p}{(2\pi)^3} e^{-ip \cdot x} S(p; \xi). \quad (4)$$

The LKFT relating the coordinate space FP in the Landau gauge to the one in an arbitrary covariant gauge reads

$$S(x; \xi) = S(x; 0) e^{-ax}, \quad (5)$$

where $a = \alpha\xi/2$. The way we proceed is as follows. We start with a FP given in the Landau gauge and Fourier transform it to the coordinate space. We then apply the LKFT law. Fourier transform of this result back to the momentum space yields the FP in an arbitrary covariant gauge.

3. LKFT of the Tree Level FP

An illustrative example to understand the usage and implications of the LKFT is to start from the lowest order fermion propagator in the Landau gauge.

$$F(p; 0) = 1 \text{ and } \mathcal{M}(p; 0) = m. \quad (6)$$

After performing the the LKFT through the procedure outlined before [8], we find that :

$$F(p; \xi) = -\frac{a}{p} \arctan \left[\frac{p}{m+a} \right] + \frac{8p(p^2 + a^2)}{\phi(p; \xi)} - \frac{8a(p^2 + a(m+a))}{\phi(p; \xi)} \arctan \left[\frac{p}{m+a} \right], \quad (7)$$

$$\mathcal{M}(p; \xi) = \frac{8p^3m}{\phi(p; \xi)}, \quad (8)$$

where

$$\phi(p; \xi) = 8p(p^2 + a(m+a)) - 8a(p^2 + (m+a)^2) \arctan \left[\frac{p}{m+a} \right]. \quad (9)$$

In the weak coupling, we can expand out Eqs. (7,8) in powers of α . To $\mathcal{O}(\alpha)$, we find

$$F(p; \xi) = 1 + \frac{\alpha\xi}{2p^2} [(m^2 - p^2)I(p) - mp^2], \quad (10)$$

$$\mathcal{M}(p; \xi) = m \left[1 + \frac{\alpha\xi}{2p^2} \{ (m^2 + p^2)I(p) - mp^2 \} \right], \quad (11)$$

where

$$I(p) = \frac{1}{p} \arctan \left[\frac{p}{m} \right]. \quad (12)$$

Let us compare these results with the one loop results obtained in [7] :

$$F_{11}(p; \xi) = 1 + \frac{\alpha \xi}{2p^2} [(m^2 - p^2)I(p) - mp^2], \quad (13)$$

$$\mathcal{M}_{11}(p; \xi) = m \times \left[1 + \frac{\alpha}{2p^2} \{ [\xi(m^2 + p^2) + 4p^2]I(p) - \xi mp^2 \} \right]. \quad (14)$$

The subscript 11 indicates that the quantities evaluated are at the one loop level. We see that the results obtained from the LKFT of a tree level FP are in accordance with the one loop FP upto a term which does not vanish in Landau gauge. Therefore, the knowledge of the lowest order FP, in conjunction with the LKFT, is sufficient to know all the gauge dependent pieces of the FP at the one loop level. The structure of the LKFT is such that the FP of order $\mathcal{O}(\alpha^n)$ in the Landau gauge fixes the coefficients of all the terms of the form $\alpha^{i+n} \xi^i$ for $i = 0, 1, \dots$ in addition to the ones of higher power in ξ at a given order in α .

4. DMG in Rainbow Approximation

In the rainbow approximation, one can write the SDE for the FP as :

$$\frac{1}{F(p; \xi)} = 1 - \frac{\alpha \xi}{\pi p^2} \int_0^\infty dk k^2 K_F(k; \xi) \times \left[1 - \frac{k^2 + p^2}{2kp} \ln \left| \frac{k+p}{k-p} \right| \right], \quad (15)$$

$$\frac{\mathcal{M}(p; \xi)}{F(p; \xi)} = \frac{\alpha(\xi + 2)}{\pi p} \int_0^\infty dk k K_M(k; \xi) \times \ln \left| \frac{k+p}{k-p} \right|, \quad (16)$$

with

$$K_F(p; \xi) = \frac{F(p; \xi)}{p^2 + \mathcal{M}^2(p; \xi)} = \frac{K_M(p; \xi)}{\mathcal{M}(p; \xi)}. \quad (17)$$

Owing to the fact that in the Landau gauge, $F(p; 0) = 1$, it has long served as a favourite gauge for the numerical study of these equations. In this gauge, one only has to solve the following equation :

$$\mathcal{M}(p; 0) = \frac{2\alpha}{\pi p} \int_0^\infty dk k K_M(k; 0) \ln \left| \frac{k+p}{k-p} \right|. \quad (18)$$

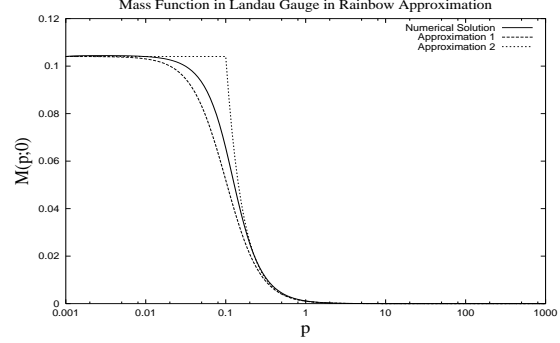


Figure 1. The mass function in the Landau gauge for the bare vertex. Approximations proposed in Eq. (19) and Eq. (20) are also shown.

The corresponding numerical solution has the following key features : It behaves like a constant for low- p and falls as $1/p^2$ for large momentum [4]. We can therefore approximate this behavior with simple analytical functions and perform the LKFT exercise to see if those key features remain intact or get modified. We can approximate the numerical solution by the following function :

$$\mathcal{M}(p; 0) = \frac{M_0 m_0^2}{p^2 + m_0^2}. \quad (19)$$

With this approximation, we find that the LKFT exercise is still not trivial to perform. Therefore, we need a further simplification. We can write another approximation to the mass function as follows :

$$\mathcal{M}(p; 0) = M_0 \left[\theta(m_0 - p) + \frac{m_0^2}{p^2} \theta(p - m_0) \right]. \quad (20)$$

As shown in Fig. (1), Eqs. (19,20) provide a good approximation. We are now in a position to use LKFT to find the fermion propagator in an arbitrary covariant gauge.

4.1. LKFT for the FP in Rainbow Approximation

The LKFT exercise for the FP in rainbow approximation can be performed, [9], to analyse the dynamically generated behaviour of the FP in the asymptotic limits of momenta, i.e., when $p \gg m_0$ and $m_0 \gg p$. In the large- p limit, the

mass function and the wavefunction renormalization have been found to have the following form :

$$\mathcal{M}(p; \xi) = \frac{C_3(\xi)}{p^2} + \mathcal{O}\left(\frac{1}{p^3}\right), \quad (21)$$

$$F(p; \xi) = 1 + \mathcal{O}\left(\frac{1}{p}\right), \quad (22)$$

where $C_3(\xi)$ is given in Ref. [9]. An analogous analysis for the low- p regime yields :

$$\mathcal{M}(p; \xi) = \frac{C_1(\xi)}{C_2(\xi)} + \mathcal{O}(p^2), \quad (23)$$

$$F(p; \xi) = -\frac{C_1^2(\xi)}{C_2(\xi)} - C_2(\xi)p^2 + \mathcal{O}(p^4). \quad (24)$$

$C_1(\xi)$ and $C_2(\xi)$ are also given in Ref. [9]. Expectedly, \mathcal{M} is flat for small values of p , and it falls off as $1/p^2$ for its large values. On the other hand, F is also a constant for small values of p . For the large values, it approaches 1. Thus the p -dependence of the dynamically generated FP for the small and large values of the momentum continues to have the same qualitative features in an arbitrary covariant gauge as the ones in the Landau gauge and in its neighbourhood, [4].

4.2. DMG with FV

As we mentioned early, a sophisticated *ansatz* for the FV must be used into the SDE instead of considering the BV. The latest in a series of proposals for the FV in QED3 is the one suggested in [7]. However, its employment to solve the SDE for the FP is a formidable task even in the Landau gauge. Let us concentrate on all vertices whose transverse part vanishes in Landau gauge, e.g., [10]. In this case, the numerical behaviour of F modifies to the one shown in Fig. (2). It behaves like a constant (different from unity) for low momentum, and tends to one as $p \rightarrow \infty$. Therefore, in addition to using the approximation in Eq. (20) for \mathcal{M} (with $M_0 \rightarrow M_{0F}$ and $m_0 \rightarrow m_{0F}$ because the solutions in the Landau gauge for the BV and the FV are not identical), we can use the following simple form for $F(p; 0)$, Fig. (2) :

$$F(p, 0) = F_{0F}\theta(m_{0F} - p) + \theta(p - m_{0F}). \quad (25)$$

The LKFT exercise is rather straightforward [9], since it involves only slight modifications to the

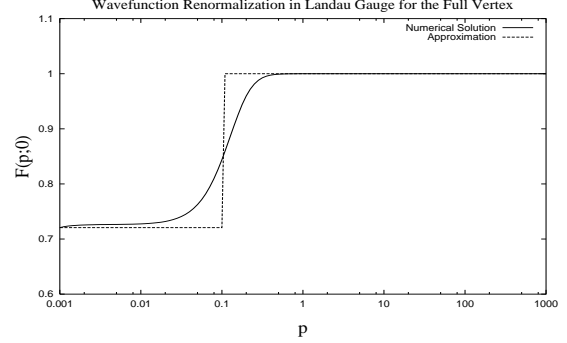


Figure 2. $F(p; 0)$ for the FV. Approximation proposed in Eq. (25) is also shown.

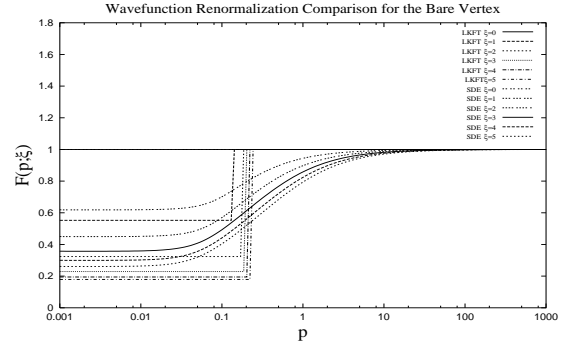


Figure 3. $F(p; \xi)$ for the BV employing LKFT. For a comparison, we also plot the results obtained by directly solving SDE.

coefficients $C_1(\xi)$, $C_2(\xi)$ and $C_3(\xi)$ and the large- and low- p behaviour of the FP remains essentially unchanged. $\mathcal{M}(p; \xi)$ behaves like a constant for low- p and falls as $1/p^2$ in an arbitrary covariant gauge. $F(p; \xi)$ is a constant for low- p and goes to 1 as p reaches a large number. These modifications are taken into account in the modified expressions for $C_{1F}(\xi)$, $C_{2F}(\xi)$ and $C_{3F}(\xi)$ given also in Ref. [9].

5. Numerical Findings

From the corresponding expressions in the low and large momentum regimes for F and \mathcal{M} , we perform the following parametrisation for the FP

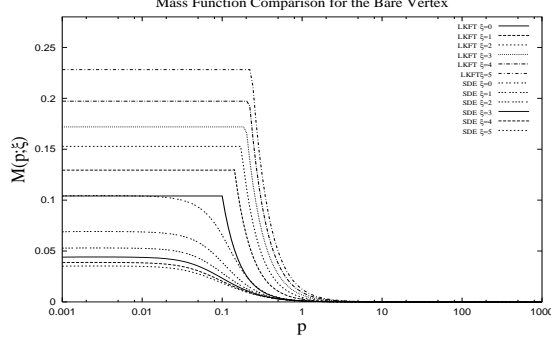


Figure 4. $\mathcal{M}(p; \xi)$ for the BV employing LKFT. For a comparison, we also plot the results obtained by directly solving SDE.

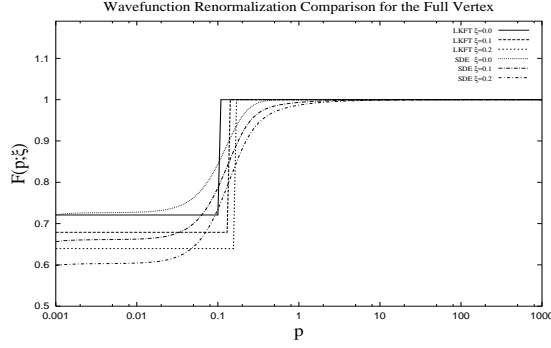


Figure 5. $F(p; \xi)$ for the FV employing LKFT. For a comparison, we also plot the results obtained by directly solving SDE.

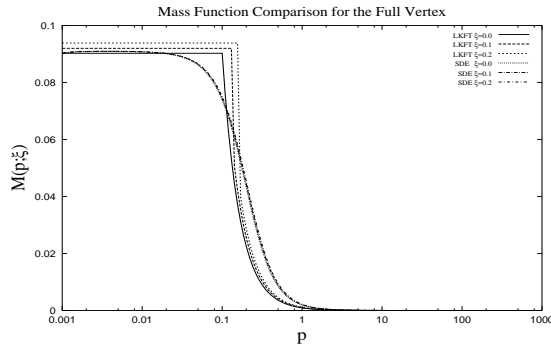


Figure 6. $\mathcal{M}(p; \xi)$ for the FV employing LKFT. For a comparison, we also plot the results obtained by directly solving SDE.

in arbitrary gauge :

$$\mathcal{M}(p; \xi) = M_{\xi(F)} \left[\theta_1 + \frac{m_{\xi(F)}^2}{p^2} \theta_2 \right], \quad (26)$$

$$F(p; \xi) = F_{\xi(F)} \theta_1 + \theta_2, \quad (27)$$

where $\theta_1 = \theta(m_{\xi(F)} - p)$, $\theta_2 = \theta(p - m_{\xi(F)})$. Moreover,

$$M_{\xi(F)} = \frac{C_{1(F)}(\xi)}{C_{2(F)}(\xi)}, \quad M_{\xi(F)} m_{\xi(F)}^2 = C_{3(F)}(\xi),$$

$$F_{\xi(F)} = -\frac{C_{1(F)}^2(\xi)}{C_{2(F)}(\xi)}. \quad (28)$$

- In Fig. (3), we have plotted $F(p; \xi)$ in several gauges. Comparing these graphs with the ones obtained by solving SDE with the BV *ansatz*, one sees that the difference is not enormous, reassuring the correctness of the method employed.
- In Fig. (4), we have plotted $\mathcal{M}(p; \xi)$ in several gauges in order to compare the results of directly solving SDE against the ones obtained by employing the LKFT.
- In Fig. (5) we present a comparison of $F(p; \xi)$ from the LKFT exercise against the results of directly solving SDE with the FV. The results are found to be in fairly good agreement.
- In Fig. (6) we present a comparison of $\mathcal{M}(p; \xi)$ from the LKFT exercise against the results of directly solving SDE with the FV. It is a bit hard to make the comparison since the SDE results are known only in a small region near the Landau gauge [4,11]. A huge advantage of using LKFT method is that the results for an arbitrary value of the covariant gauge parameter are readily available.

6. Conclusions and Outlook

Knowledge of the FP at an arbitrary order in perturbation theory in an arbitrary covariant gauge is as useful as it is difficult. If we are dealing with perturbative calculations, the number of diagrams increases enormously as the order of approximation gets bigger. Since the LKFT of a given order FP already fixes some of the coefficients of the FP in its all orders expansion, it is worth making use of it. If we are dealing with

nonperturbative studies of SDE, where it is hard to obtain solutions for arbitrarily large values of the gauge parameter, LKFT comes to rescue once again. If we are capable of solving the SDE for the FP with a sophisticated *ansatz* for the FBV in Landau gauge, its LKFT yields the dynamically generated FP for an arbitrarily large value of the gauge parameter. We expect this procedure to be translated to more complicated theories, like QCD, with only slight modifications.

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